DERIVADAS

Definición

$$\frac{df(x)}{dx} = \lim_{\Delta\to 0}\frac{\Delta f(x)}{\Delta x} = \lim_{\Delta\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

Generalidades

1. $\frac{d}{dx}(c) = 0$ — Derivada de una Constante
2. $\frac{d}{dx}(cx) = c$
3. $\frac{d}{dx}(cx^n) = ncx^{n-1}$
4. $\frac{d}{dx}(u \pm v \pm w \pm \cdots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \pm \cdots$ — Derivada de la suma de Funciones de x
5. $\frac{d}{dx}(cu) = c\frac{du}{dx}$
6. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
7. $\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}$ — Derivada del producto de funciones de x
8. $\frac{d}{dx}\frac{u}{v} = \frac{v(du/dx)-u(du/dx)}{v^2}$ — Derivada del cociente de funciones de x
9. $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$
10. $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ — Regla de la cadena
11. $\frac{du}{dx} = \frac{1}{dx/du}$
12. $\frac{dy}{dx} = \frac{dy/du}{dx/du}$

Derivadas de funciones trigonométricas

1. $\frac{d}{dx}\, sen\, u = \cos u\, \frac{du}{dx}$
2. $\frac{d}{dx}\cos u = -sen\, u\, \frac{du}{dx}$
3. $\frac{d}{dx}\tan u = sec^2 u\, \frac{du}{dx}$

4. $\frac{d}{dx}\cot u = -csc^2\, u\, \frac{du}{dx}$

5. $\frac{d}{dx}\sec u = \sec u \tan u \frac{du}{dx}$

6. $\frac{d}{dx}\csc u = -\csc u \cot u \frac{du}{dx}$

Derivadas de funciones trigonométricas inversas

1- $\frac{d}{dx} arcsen\, u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ $\qquad \left\{ -\frac{\pi}{2} < arcsen\, u < \frac{\pi}{2} \right.$

2- $\frac{d}{dx}\ arcos\ u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$ $\qquad \left\{ 0 < arccos\ u < \pi \right.$

3- $\frac{d}{dx}\arctan u = \frac{1}{1+u^2}\frac{du}{dx}$ $\qquad \left\{ -\frac{\pi}{2} < arctan\ u < \frac{\pi}{2} \right.$

4- $\frac{d}{dx}\text{arccot}\, u = \frac{1}{1+u^2}\frac{du}{dx}$ $\qquad \left\{ 0 < arccot\ u < \pi \right.$

5- $\frac{d}{dx}\ arcsen\ u = \frac{1}{|u|\sqrt{u^2-1}}\frac{dy}{dx} = \frac{\pm 1}{u\sqrt{u^2-1}}\frac{du}{dx}$ $\qquad \begin{cases} +\ si\ 0 < \text{arcsec}\, u < \pi/2 \\ -\ si\ \frac{\pi}{2} < \text{arcsec}\, u < \pi \end{cases}$

6- $\frac{d}{dx}\ arcsc\ u = \frac{-1}{|u|\sqrt{u^2-1}}\frac{dy}{dx} = \frac{\mp 1}{u\sqrt{u^2-1}}\frac{du}{dx}$ $\qquad \begin{cases} -\ si\ 0 < \text{arcsc}\, u < \pi/2 \\ +\ si\ \frac{\pi}{2} < \text{arcsc}\, u < \pi \end{cases}$

Derivadas de funciones logarítmicas y exponenciales

1- $\frac{d}{dx}\log_a u = \frac{\log_a e}{u}\frac{du}{dx}$

2- $\frac{d}{dx}\ln u = \frac{d}{dx}\log_e u = \frac{1}{u}\frac{du}{dx}$

3- $\frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}$

4- $\frac{d}{dx}e^u = e^u \frac{du}{dx}$

5- $\frac{d}{dx}\ u^v = \frac{d}{dx}\ e^{v \ln u} = e^{v \ln u}\frac{d}{dx}(v \ln u) = vu^{v-1}\frac{du}{dx} + u^v \ln u \frac{dv}{dx}$

Derivadas de funciones Hiperbólicas

1- $\frac{d}{dx} senh\, u = \cosh u \frac{du}{dx}$

2- $\frac{d}{dx} \cosh u = senh\, u \frac{du}{dx}$

3- $\frac{d}{dx} \tanh u = sech^2 u \frac{du}{dx}$

4- $\frac{d}{dx} \coth u = -csch^2 u \frac{du}{dx}$

5- $\frac{d}{dx} \operatorname{sech} u = -\operatorname{sech} u \tanh u \frac{du}{dx}$

6- $\frac{d}{dx} \operatorname{csch} u = -\operatorname{csch} u \coth u \frac{du}{dx}$

Derivadas de funciones hiperbólicas inversas

1- $\frac{d}{dx} argsenh\, u = \frac{1}{\sqrt{u^2+1}} \frac{du}{dx}$

2- $\frac{d}{dx}\ argcosh = \frac{\pm 1}{\sqrt{u^2-1}} \frac{du}{dx}$ $\begin{cases} +\ si\ argcosh\, u > 0, u > 1 \\ -\ si\ argcosh\, u < 0, u > 1 \end{cases}$

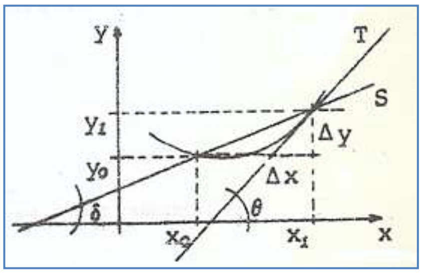
3- $\frac{d}{dx}\ argtanh\, u = \frac{1}{1+u^2} \frac{du}{dx}$ $\{-1 < u < 1$

4- $\frac{d}{dx}\ argcoth\, u = \frac{1}{1-u^2} \frac{du}{dx}$ $\{u > 1 \text{ ó } u < -1$

5- $\frac{d}{dx}\ argsech\, u = \frac{\mp 1}{u\sqrt{1-u^2}} \frac{du}{dx}$ $\begin{cases} -\ si\ argsech\, u > 0\ y\ 0 < u < 1 \\ +\ si\ argsech\, u < 0\ y\ 0 < u < 1 \end{cases}$

6- $\frac{d}{dx} argcsch\, u = \frac{-1}{|u|\sqrt{1+u^2}} \frac{du}{dx} = \frac{\mp 1}{u\sqrt{1+u^2}} \frac{du}{dx}$ $\begin{cases} -\ si\ \ u > 0 \\ +\ si\ \ u < 0 \end{cases}$

Interpretación Gráfica de la función Derivación



$y = f(x)$, $y_0 = f(x_0)$, $y_1 = f(x_1) = f(x_0 + \Delta x)$

$\Delta y = \Delta f(x) = \Delta f(x_1) - \Delta f(x_0)$

$\frac{\Delta y}{\Delta x} = \tan \delta \cong [recta\ S]$

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \tan \theta \equiv [recta\ T] = f'(x)$

$f'(x) = derivada\ de\ f(x) = pendiente\ de\ la\ recta\ tangente\ a\ la\ curva\ de\ la\ función\ en\ un\ punto.$

Tabla de Integrales Indefinidas

Reglas para una integración

Nota: a, b, p, q, n son constantes; u, v, w, son funciones de x

1- $\int a\,dx = ax$

2- $\int af(x)dx = a\int f(x)\,dx$

3- $\int (u \pm v \pm w \pm \cdots)dx = \int udx \pm \int vdx \pm \ldots$

4- $\int udv = uv - \int vdu$

5- $\int f(ax)dx = \frac{1}{a}\int f(u)du$

6- $\int F\{f(x)\}dx = \int F(u)\frac{dx}{du}du = \int \frac{F(u)}{f'(x)}du$

7- $\int u^n du = \frac{u^{n+1}}{n+1}, n \neq -1 \quad Para\ n = -1, ver\ (8)$

8- $\int \frac{du}{u} = \ln u = \ln|u| \quad si \quad u > 0 \quad o \quad \ln(-u) \quad si\ u < 0$

9- $\int e^u du = e^u$

10- $\int a^u du = \int e^{u\ln a}du = \frac{e^{u\ln a}}{\ln a} = \frac{a^u}{\ln a}; a > 0; a \neq 1$

11- $\int sen\,u\,du = -\cos u$

12- $\int \cos u\,du = sen\,u$

13- $\int \tan u\,du = \ln\sec u = -\ln\cos u$

14- $\int \cot u\,du = \ln\sec u$

15- $\int \sec u\,du = \ln(\sec \mathrm{u} + \tan \mathrm{u}) = \ln\tan\left(\frac{\mathrm{u}}{2} + \frac{\pi}{4}\right)$

16- $\int \csc u\,du = \ln(\csc u - \cot u) = \ln\tan\frac{u}{2}$

17- $\int sec^2\,u\,du = \tan u$

18- $\int csc^2\,u\,du = -\cot u$

19- $\int tan^2\,u\,du = \tan u - u$

20- $\int cot^2\,u\,du = -\cot u - u$

21- $\int sen^2\,u\,du = \frac{u}{2} - \frac{sen\,2u}{4} = \frac{1}{2}(u - sen\,u\cos u)$

22- $\int cos^2\,u\,du = \frac{u}{2} + \frac{sen\,2u}{4} = \frac{1}{2}(u + sen\,u\cos u)$

23- $\int \sec u\tan u\,du = \sec u$

24- $\int \csc u\cot u = -\csc u$

25- $\int senh\,u\,du = \cosh u$

26- $\int \cosh u\,du = senh\,u$

27- $\int \tanh u \, du = \ln \cosh u$

28- $\int \coth u \, du = \ln senh \, u$

29- $\int \operatorname{sech} u \, du = sen^{-1}(\tanh u)$ ó $2tan^{-1}e^u$

30- $\int \operatorname{csch} u \, du = \ln \tanh \frac{u}{2}$ ó $- coth^{-1}e^u$

31- $\int sech^2 u \, du = \tanh u$

32- $\int csch^2 u \, du = -\coth u$

33- $\int tanh^2 u \, du = u - \tanh u$

34- $\int coth^2 \, u \, du = u - \coth u$

35- $\int senh^2 \, u \, du = \frac{senh \, 2u}{4} - \frac{u}{2} = \frac{1}{2}(senh \, u \cosh u - u)$

36- $\int cosh^2 \, u \, du = \frac{senh \, 2u}{4} + \frac{u}{2} = \frac{1}{2}(senh \, u \cosh u + u)$

37- $\int \operatorname{sech} u \tanh u \, du = -sech \, u$

38- $\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u$

39- $\int \frac{du}{u^2+a^2} = \frac{1}{a} tan^{-1} \frac{u}{a}$

40- $\int \frac{du}{u^2-a^2} = \frac{1}{a} \ln(\frac{u-a}{u+a}) = -\frac{1}{a} coth^{-1} \frac{u}{a} \quad u^2 > a^2$

41- $\int \frac{du}{a^2-u^2} = \frac{1}{2a} \ln(\frac{a+u}{a-u}) = -\frac{1}{a} tanh^{-1} \frac{u}{a} \quad u^2 < a^2$

42- $\int \frac{du}{\sqrt{a^2-u^2}} = sen^{-1} \frac{u}{a}$

43- $\int \frac{du}{\sqrt{u^2+a^2}} = \ln\left(u + \sqrt{u^2+a^2}\right)$ ó $senh^{-1} \frac{u}{a}$

44- $\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left(u + \sqrt{u^2-a^2}\right)$

45- $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} sec^{-1} \left|\frac{u}{a}\right|$

46- $\int \frac{du}{u\sqrt{u^2+a^2}} = -\frac{1}{a} \ln(\frac{a+\sqrt{u^2+a^2}}{u})$

47- $\int \frac{du}{u\sqrt{u^2-a^2}} = -\frac{1}{a} \ln(\frac{a+\sqrt{a^2-u^2}}{u})$

48- $\int f^{(n)} g \, dx = f^{(n-1)} g - f^{(n-2)} g' + f^{(n-3)} g'' - \cdots (-1) \int f g^{(n)} dx$

Método de Sustitución

49- $\int F(ax+b)dx = \frac{1}{a} \int F(u)du \qquad u = ax+b$

50- $\int F\left(\sqrt{ax+b}\right) dx = \frac{2}{a} \int uF(u)du \qquad u = \sqrt{ax+b}$

51- $\int F\left(\sqrt[n]{ax+b}\right)dx = \frac{n}{a} \int u^{n-1} F(u)du \quad u = \sqrt[n]{ax+b}$

52- $\int \left(\sqrt{a^2-x^2}\right)dx = a \int F(a\cos u) \cos u \, du \qquad x = a \, sen \, u$

53- $\int \left(\sqrt{x^2 + a^2}\ \right) dx = a \int F(a\ sen\ u) \sec^2 u\, du$ $\qquad x = a \tan u$

54- $\int F\left(\sqrt{x^2 - a^2}\right) dx = a \int F(a \tan u) \sec u \tan u\, du$ $\qquad x = a \sec u$

55- $\int F(e^{ax}) dx = \frac{1}{a} \int \frac{F(u)}{u} du$ $\qquad u = e^{ax}$

56- $\int F(\ln x) dx = \int F(u) e^u du$ $\qquad u = \ln x$

57- $\int F\left(sen^{-1} \frac{x}{a}\right) dx = a \int F(u) \cos u\, du$ $\qquad u = sen^{-1} \frac{x}{a}$

58- $\int F(sen\ x, \cos c) dx = 2 \int F\left(\frac{2u}{1+u^2}, \frac{1-u^2}{1+u^2}\right) \frac{du}{1+u^2}$ $\qquad u = \tan \frac{x}{2}$